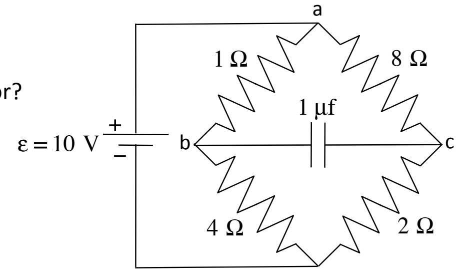
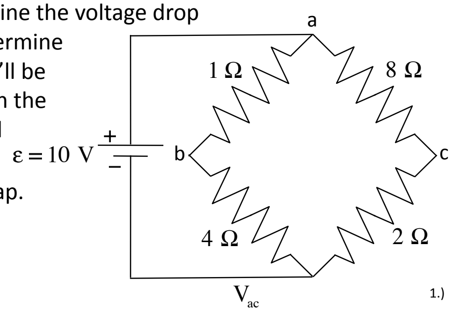
Problem 28.37

a.) Voltage difference across capacitor?

This is fun! When fully charged, there is no current through the capacitor. That means the circuit could be re-drawn as shown below.



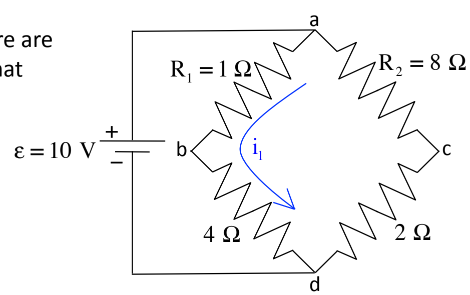
We know the electrical potential of Point "a" is 10 volts (it's connected to the high side of the battery). If we can determine the voltage drop across the 1 ohm resistor, we can determine the electrical potential at Point "b" (it'll be 10 - ΔV_{a-b}). Doing a similar process on the other side and we have what we need to determine the voltage across b-c, which will be the voltage across the cap.



To that end:

With the cap out of commission, there are 10 volts across Points "a" and "d." That means we can write:

$$10\text{ V} = (i_1)(1\ \Omega + 4\ \Omega)$$
$$\Rightarrow \quad i_1 = 2\text{ A}$$



With i_1, we can write:

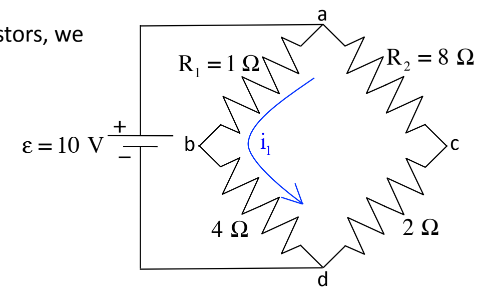
$$\Delta V_{a-b} = i_1 R_1$$
$$= (2\text{ A})(1\ \Omega)$$
$$= 2\text{ V}$$

That means the electrical potential at Point "b" is:

$$V_b = V_a - V_{drop}$$
$$= (10\text{ V}) - (2\text{ V})$$
$$\Rightarrow \quad V_b = 8\text{ A}$$

Similarly for the right section of resistors, we can write:

$$10\text{ V} = (i_2)(8\ \Omega + 2\ \Omega)$$
$$\Rightarrow \quad i_2 = 1\text{ A}$$



With i_2, we can write:

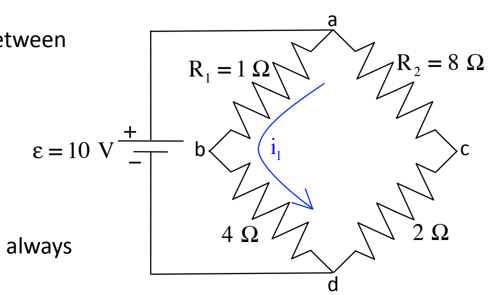
$$\Delta V_{a-c} = i_2 R_2$$
$$= (1\text{ A})(8\ \Omega)$$
$$= 8\text{ V}$$

That means the electrical potential at Point "c" is:

$$V_c = V_a - V_{drop}$$
$$= (10\text{ V}) - (8\text{ V})$$
$$\Rightarrow \quad V_c = 2\text{ A}$$

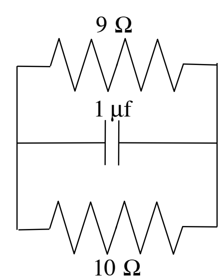
Apparently, the net voltage drop between Points "b" and "c" is:

$$V_{b-c} = V_c - V_b$$
$$= (2\text{ V}) - (8\text{ V})$$
$$\Rightarrow \quad V_{cap} = 6\text{ V}$$



(as the voltage across a capacitor is always defined as positive)

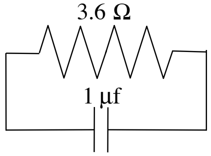
b.) With the battery disconnected, the circuit becomes as shown to the right.



If we combine the resistor in parallel, we get an equivalent resistance of:

$$R = \left[\frac{1}{9\ \Omega} + \frac{1}{6\ \Omega}\right]^{-1}$$
$$= 3.6\ \Omega$$

So our circuit is now as shown to the right.



The time constant for that circuit is:

$$\tau = RC$$
$$= (3.6\ \Omega)(1\text{x}10^{-6}\ \text{f})$$
$$= 3.6\text{x}10^{-6}\ \text{seconds}$$

The voltage across the capacitor will be the same as the voltage across the resistor. As the voltage across the resistor is a function of the current through it, and as the current is governed by the relationship:

$$e^{-t/RC}$$

The amount of time required for that factor to drop to 1/10 is:

$$e^{-t/RC} = \frac{1}{10}$$
$$\Rightarrow \ln\left(e^{-t/RC}\right) = \ln\left(\frac{1}{10}\right)$$
$$\Rightarrow -t/RC = \ln\left(\frac{1}{10}\right)$$
$$\Rightarrow t = -RC\ln\left(\frac{1}{10}\right)$$
$$= -(3.6\ \Omega)(10^{-6}\ \text{f})(-2.3)$$
$$= 8.28\text{x}10^{-6}\ \text{seconds}$$

Note: Remember that after two time constants, the current will drop to 13.5% of the maximum current and voltage (just a hint above our 10% mark). Twice our time constant in this instance is:

$$2\tau = 2RC$$
$$= 2(3.6\text{x}10^{-6}\ \text{seconds})$$
$$= 7.2\text{x}10^{-6}\ \text{seconds}$$

As expected, this is a little shy of the value of $8.28\text{x}10^{-6}$ seconds that we calculated.